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Three order state space modeling of proton exchange membrane fuel cell with energy function definition

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ABSTRACT

The fuel cell is a complex system which is the centre of a lot of multidisciplinary research activities since it involves intricate application of various fields of study. The operation of a fuel cell depends on a wide range of parameters. The effect of one cannot be studied in isolation without disturbing the system which makes it very difficult to comprehend, analyze and predict various phenomena occurring in the fuel cell. In the current work, we present an equivalent electrical circuit of the pneumatics and fluidics in a fuel cell stack. The proposed model is based on the physical phenomena occurring inside fuel cell stack where we define the fluidic-electrical and pneumatic-electrical analogy. The effect of variation in temperature and relative humidity on the cell are considered in this model. The proposed model, according to the considered hypothesis, is a simple three order state space model which is suitable for the control purpose where a desired control structure can be formulated for high-end applications of the fuel cell as a subpart of a larger system, for instance, in hybrid propulsion of vehicles coupled with batteries and supercapacitors. Another key point of our work is the definition of the natural fuel cell stack energy function. The circuit analysis equations are presented and the simulated model is validated using the experimental data obtained using the fuel cell test bench available in Fuel Cell Laboratory, France.

1. Introduction

As the world is becoming technologically dependent, more energy is being used to keep up with the changing requirements. At the current rate at which energy is being used, we will run out of fossil fuels sooner rather than later. In addition, the global concern over the harmful emissions by their combustion has been a major cause of development of alternative methodologies for meeting our energy requirements. A fuel cell is an energy conversion device that converts the chemical energy of a fuel directly into electricity. Energy is released whenever a fuel (hydrogen) reacts chemically with the oxygen of air. The reaction occurs electrochemically and the energy is released as a combination of low-voltage DC electrical energy and heat. Today, the existence of fuel cells with different working characteristics, allows fuel cell technology to be adapted to most of our power needs, for instance, in static generators or embedded power sources for transport applications.

The fuel cell is a complex system involving many fields of study like fluid mechanics, mass-transfer, electrochemistry and thermodynamics. There are so many parameters involved, which can all not be measured without disturbing the system, that it makes many phenomena occurring in the fuel cell difficult to predict or even comprehend. Over the years, different approaches have been used by scientists to model this complex system. Nguyen [1] face the problem by estimating products and reactants concentration in electrode and cathode channels, while laying special emphasis on water transport phenomenon. Their model determines the reactant/product concentrations in the channel using partial derivatives over channel length. There are many detailed models of fuel cell available in literature, however, the high level of specificity and complexity does not permit their use by electrical engineers in the range of applications. The model presented in the current work is developed to represent the behaviour of different species in the fuel cell channels in the form of an electrical equivalent model. It intends to depict the fuel cell system with the simplicity of an electrical circuit, so that it can be easily understood and conveniently used by electrical engineers in a gamut of applications involving simulation processes concerning dynamic behaviour of the fuel cell systems.

Other analytical model of FC with the moto-compressor was recently published in [2,3]. This model is very interesting since

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it is only 4 order state space model, but the developed reduced order model is not valid in a FC starting period. So authors in those papers propose to start FC with another (complete order model), then switch to the reduced one. The model presented in this paper is a FC model of only three states order and is validated in all FC functioning region including FC startup.

The following sections in this paper present a brief review of different existing electrical equivalent models of fuel cell followed by the introduction to our model under the stated assumptions and analogies. We then present the electrical equivalent circuit thus formulated and develop the equations pertaining to this circuit. And finally we compare the simulations and the experimental data that we obtain using MATLAB Simulink environment.

2. Fuel cell description

Scientists today are working on a variety of fuel cells. There are many types of fuel cells, different types catering to a different operating condition. The work presented in this text is primarily concerned with the modeling of a hydrogen fuel cell which is an electrochemical device which uses hydrogen as a fuel and oxygen as an oxidant. These entities are subject to electrochemical reactions on either side of a cation conducting membrane which acts as an electrolyte, hence producing an electric current without involving any combustion, thereby, limiting the heat losses and yielding an efficient (considerable improvement if compared to thermal machines) conversion of energy. This process involves generation of water as the only by-product which is very much more desirable than any green house gas emissions from combustion.

The air and hydrogen are injected onto the catalytic porous electrode plates, nowadays different technologies like metallic foams or canals over the plates are used in order to ensure a good gas distribution on catalytic sites. In order to provide an adequate gas management, several additional ancillary circuits need to be a part of the fuel cell system like the hydrogen circuit, cooling circuit and humidification circuit as shown in Fig. 1.

The hydrogen circuit can be operated in three modes, open-mode, closed-mode and feedback mode. This circuit controls the pressure, mass/volume flow rates, temperature and vapor content of hydrogen. The open mode operation involves the remainder of hydrogen (i.e. the hydrogen left after passing through the anode) being released to the water condenser used by the cooling system. In the closed mode operation, the hydrogen exit valve closes; however, it is opened only to ensure hydrogen renovation by a process called “flush”. In the feedback mode, the remainder hydrogen is re-injected into the cell.

The air (oxidant) is humidified prior to its introduction in the FC stack. The humidification process involves heating the air and humidifying it at a high temperature and then cooling this mixture to a lower temperature so as to get a saturated mix of water vapor and air at this temperature, which is measured by the sensors.

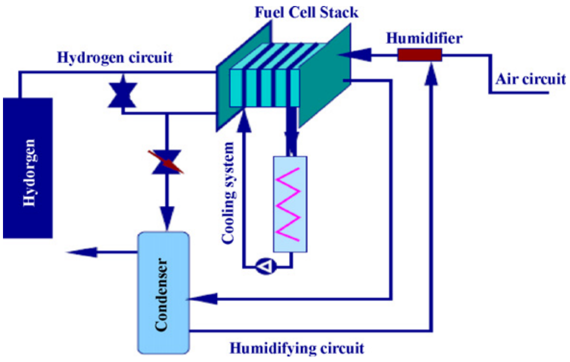

Fig. 1. Fuel cell description.

3. Equivalent electrical circuit

In the current text, an application based model of the fuel cell by taking into consideration all necessary parameters which are key to FC operation and carefully avoiding the intricate details. The modeling approach based on the principles of conservation of mass, charge, momentum and energy.

The equivalent electrical model approach aims at establishing a simple and reliable model of the fuel cell. Scientists over the years have presented many electrical equivalent circuit models of fuel cell. To get a brief overview of the electrical equivalent models of fuel cell, one is advised to go through Hernandez et al. [4] and Runtz and Lyster [5].

3.1. The modeling approach

The electrical equivalent model of a fuel cell is obtained by setting an equivalent electrical component to account for the pneumatic behaviour. This equivalence is based on the conservation principles of mass, charge and energy. Following the Bernoulli’s laws of continuity, mass and energy flow are conserved in a pipe for an incompressible fluid, however there will be losses due to friction and geometry of the pipes. The compressibility of the fluid will add a storing capacity to the pipes. Hence, the molar flow rates in a pneumatic system can be analogous to the flow of charge, i.e. current in an electrical system, based on the fact that mass and charge are conserved in both the systems. Similarly, the driving force for charge and mass is the electrical potential difference and pressure difference in the respective cases. The flows vary linearly with respect to the drive force, which in the electrical systems is given by the Ohms law. In pneumatic systems, an equivalent pneumatic resistance can be computed which will satisfy the analogous equation of Ohms law for the pneumatic system.

$$\text{electrical environment}: \quad I = \frac{dq}{dt}$$

$$\text{pneumatic environment}: \quad Q = \frac{dn}{dt}$$

$$Q_{m,net} = Q_{m,in} - Q_{m,out}$$

The gases that are being passed through the channels are compressible. This compressibility is determined by the volume the channel can hold under given pressure and temperature conditions. This is analogous to a capacitor in an electrical system which is capable of accumulating charge under transient conditions and acts as an open circuit when steady state is achieved. The Fig. 2 shows the analogy used to model the capacitance of channels in our circuit [4].

The system has water as one of the key species which takes part in the reaction as well as in various diffusion phenomena. Therefore, it becomes necessary to consider water in both its phases, i.e. liquid and vapor. If the partial pressure of water at a certain temperature exceeds the value of its saturation pressure (given by the Antoine’s equation for a constant value of temperature) it condenses till the value of the partial pressure becomes equal to the saturation pres-

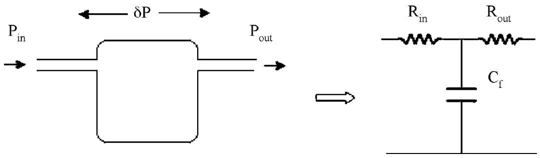

Fig. 2. Storage capacity of channel analogous to a capacitor.

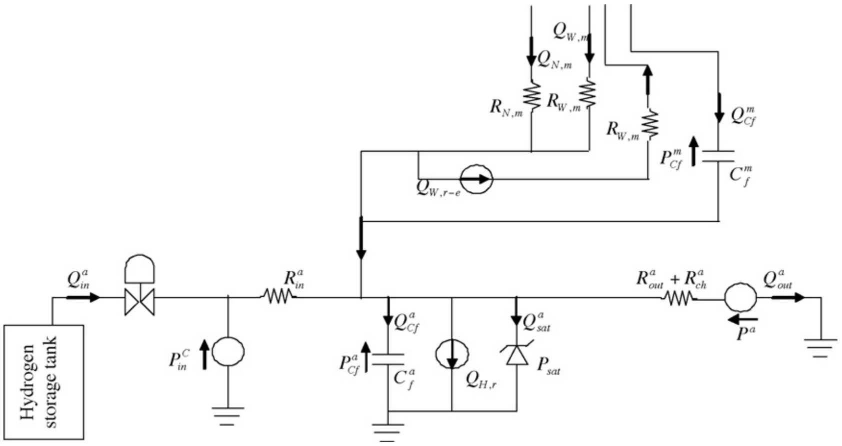

Fig. 3. Anode circuit.

sure, thereby giving us water in a liquid state in the circuit as well, which is very important to maintain the conductivity of the membrane. This saturation pressure is analogous to a zener diode in an electric system, which does not allow any current to pass through it if the potential drop across it is lesser than its saturation potential (similarly there is no liquefaction if partial pressure is less than the saturation value) and allows variable current to flow so long as the same potential drop (equal to saturation potential) is maintained. The model proposed in this work is based on some assumptions listed as follows:

- All the gases are ideal gases.
- The interaction between different species is negligible.
- The principle of superposition for electric circuits is applicable.
- The pressure changes produced by the velocity of the flow are negligible.
- The changes in the volume of channels are considered negligible.
- The flow is assumed to be laminar.
- The losses due to fuel and oxidant crossover are considered to be negligible.
- The only species that diffuse through the membrane are nitrogen and water.
- The water is introduced only on the cathode side.
- The channel pressure for anode and cathode is taken to be the average of the inlet and outlet pressures.

3.2. Anode circuit

In a fuel cell, the side undergoing oxidation and in the process releasing electrons is conventionally referred to as the anode. In our case, hydrogen acts as a fuel and undergoes oxidation at the anode. The electric equivalence established on the principles stated in the preceding section is used to develop an equivalent anode circuit shown in Fig. 3. The hydrogen enters in the channel directly from the storage tank and the inlet pressure is controlled by a valve. This hydrogen diffuses through the gas diffusion layers and reacts on the surface of the catalyst to liberate electrons which are responsible for the current we get in the external circuit, separating them from the leaving behind protons which are transported to the other side through the proton exchange membrane which is a good ionic conductor. The electrons that are generated in this reaction are used to power the load device. The rate of depletion of hydrogen is a function of the current which is observed to pass through the load resistance.

The polymer membrane is not completely impermeable to species taking part in the operation. Besides hydrogen, there is significant amount of diffusion of water and nitrogen across the membrane. This diffusion is driven by the concentration and pressure differences on either side of the membrane. If we assume the concentration difference to remain constant, we can compute an equivalent resistance to predict this crossover as a function of the pressure difference between the two sides of the membrane. There would also be a backflow of water once it crosses over because of the drag of the diffusing protons through the membrane which can force the water molecules through the membrane back to the cathode section. This is also termed as reverse electro-osmotic drag. To account for this backflow, we use a correlation given in [6] which give us the rate of flow of water due to this phenomenon. The zener diode accounts for the condensation on the anode side. The capacity of the anode channel remains the same, irrespective of the species in consideration hence, placed in parallel as seen in the circuit in fig. 3.

3.3. Cathode circuit

We introduce the air into the cathode section with a certain degree of humidification which is given by a sensor in the experimental bench. The circuit of air humidification forms a part of an auxiliary system and can be studied in more detail as is given in [8]. The humidifying circuit is not taken into consideration in the model proposed in herein. However, the effects of the variation in the inlet temperature and average channel temperature are taken into consideration. The reaction of oxygen and protons results in the generation of water, which is shown in the circuit given in Figs. 3 and 4. The water generated can also be represented as a function of current. Note the presence of the zener diode which indicates the condensation of water if its partial pressure exceeds the stipulated saturation pressure at the average temperature in the channel.

Nitrogen diffusion is not usually considered as an important factor and is generally not included in most of the existing models, since in most conditions of operation the effect of nitrogen migration is rather small. However, if the stack is set to work in a closed mode (hydrogen outlet closed) to optimize fuel consumption, a considerable amount of nitrogen can be stored on the anode side of the stack between flushes, leading to abnormal behaviour characterized by a voltage drop. It is then important to include nitrogen migration in a model which is intended to be used in future for failure analysis [9].

4. Analysis of model

The overall circuit hence formulated is shown in Fig. 5. Each of the flow rates is denoted by the letter Q. We need

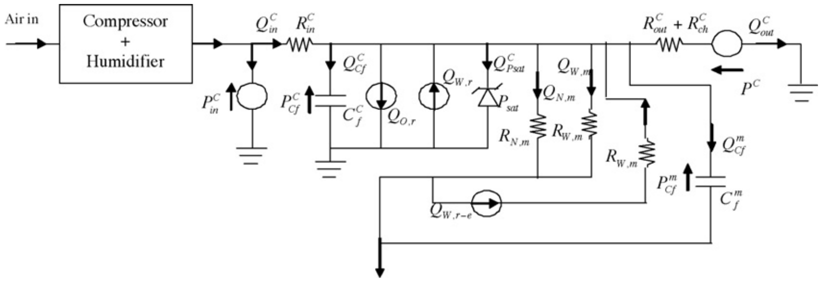

Fig. 4. Cathode circuit.

to develop the equations so as to solve for the unknown flow rates through capacitor and predict the overall potential of the stack.

4.1. Network analysis

This section presents the formulation of these equations based on Kirchhoff laws, which are then used to carry out the simulations. The reader is advised to go through Appendix A for more information on all the variables used in the equations developed in this section. The letter Q denotes the flow rate whereas the superscripts I, a, c and m stand for function of current, anodic, cathodic or membrane sections, respectively. The subscripts in first place are the first letter of the species they denote, for instance, w for water, H for hydrogen, etc., the subscript C_f denotes either the flow rate or pressure drop through the capacitor in the respective section. 'r', '$r-e$', 'm', 'in', 'out' and 'P_{sat}' denote reacted, reverse electro-osmosis, membrane, inlet, outlet and property through the zener diode section.

The following equations relate the flow rates in the different branches in the circuit shown in Fig. 5.

$$Q_{in}^c + \nu_w Q_{w,r}^I + \nu_w Q_{w,r-e}^I = \nu_o Q_{O,r}^I + Q_{P_{sat}}^c + Q_{N,m} + Q_{w,m} + Q_{out}^c \quad (1)$$

$$Q_{in}^a + Q_{N,m} + Q_{w,m} + Q_{C_f}^m = Q_{C_f}^c + \nu_w Q_{w,r-e}^I + \nu_h Q_{H,r}^I + Q_{P_{sat}}^a + Q_{out}^a \quad (2)$$

$$P_{C_f}^c = (R_{out}^c + R_{ch}^c)Q_{out}^c + P^c \quad (3)$$

$$P_{C_f}^a = (R_{out}^a + R_{ch}^a)Q_{out}^a + P^a \quad (4)$$

$$P_{in}^c = (R_{in}^c)Q_{in}^a + P_{C_f}^c \quad (5)$$

$$P_{C_f}^m = (0.5R_{w,m} + R_{out}^a + R_{ch}^a)Q_{out}^a + P^a \quad (6)$$

$$P_{C_f}^m = (0.5R_{w,m} + R_{out}^a + R_{ch}^a)Q_{out}^a + P^a \quad (7)$$

$$\frac{dP_{C_f}^c}{dt} = \frac{Q_{C_f}^c}{C_f^c} \quad (8)$$

$$\frac{dP_{C_f}^a}{dt} = \frac{Q_{C_f}^a}{C_f^a} \quad (9)$$

$$\frac{dP_{C_f}^m}{dt} = \frac{Q_{C_f}^m}{C_f^m} \quad (10)$$

$$P^{ca} = R_{N,m}Q_{N,m} \quad (11)$$

where ν_i denotes the molar volume of i where $i = \{w$ for water, o for oxygen or h for hydrogen$\}$.

4.2. Parameters and known functions

In the above equations, we notice that there are some variables which are a function of current and some which are available from the data. The estimation of parameters is key to obtain a desirable simulation results which model the behaviour of the fuel cell with minimum deviation to the experimental values. The list of

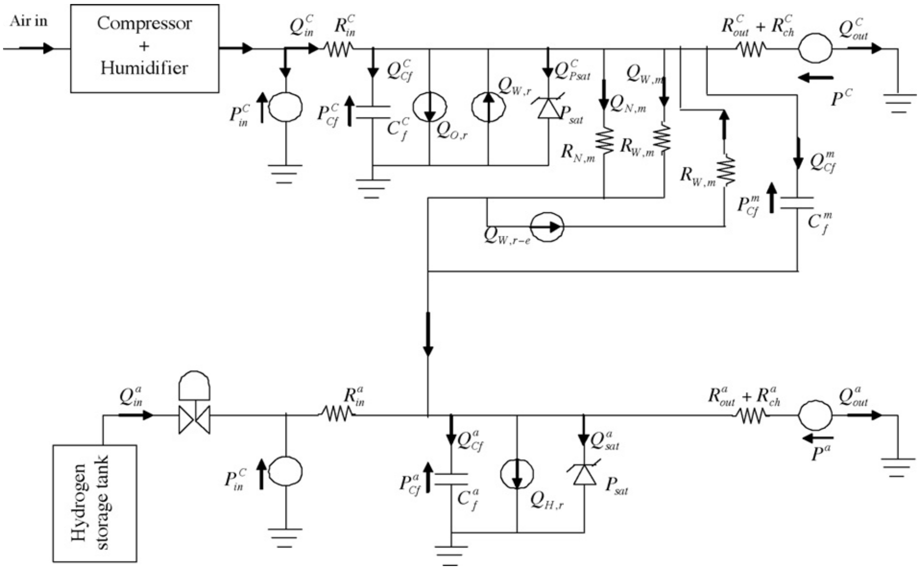

Fig. 5. Complete circuit: anode, cathode and membrane.

parameters used can be found in the doctorate thesis submitted by Hernandez at the FClab, UTBM, France [10]. At approximately 321 K the following values of parameters are proposed by Hernandez and the molar volume at the same temperature being 26 l mol^{-1}:

$$C_f^c = 130\,\mathrm{l\,mbar^{-1}}$$
$$C_f^a = 78\,\mathrm{l\,mbar^{-1}}$$
$$R_{out}^c, R_{in}^c = (0.018Q_{in}^c + 3.1)\,\mathrm{mbar\,min\,l^{-1}}$$
$$R_{out}^a, R_{in}^a = (0.683Q_{in}^c + 3.5)\,\mathrm{mbar\,min\,l^{-1}}$$

The above correlation holds under the condition that the flow rate of hydrogen is between 3 and 7 l min^{-1} and is developed by plotting pressure drops and flow rates in the respective channels and using regression on over a range of sampled values hence obtained. The details can be found in [10].

$$R_{ch}^c = (0.066Q_{out}^c + 0.79)\,\mathrm{mbar\,min\,l^{-1}}$$
$$R_{ch}^a = (0.082Q_{out}^a + 3.2)\,\mathrm{mbar\,min\,l^{-1}}$$
$$R_{N,m} = 31.550\,\mathrm{mbar\,min\,l^{-1}}$$
$$R_{N,m} = 1.26\,\mathrm{mbar\,min\,l^{-1}}$$

There are some flow rates which can be calculated directly as functions of the measured current during the operation. The functions are presented below:

$$Q_{H,r}^I = Q_{O,r}^I = \frac{I}{2F}$$
$$Q_{w,r}^I = \frac{I}{4F}$$

To get the water content inside the membrane we refer to [6]. Note that the water content at 303 K and 353 K are given and the water content at the given temperature between these two limits can be calculated by linear interpolation. To calculate the water content we need to get the value of activities which can be easily calculated as they depend on relative humidity values which are available in the experimental data. As per the phenomenological model of fuel cell (developed by Springer) the flow of water through the membrane as a result of reverse osmosis phenomenon is given by the following correlation:

$$Q_{w,r-e}^I = n_d \frac{I}{F} \tag{12}$$

where, n_d is a function of the water content of the membrane given by the relation [6].

$$n_d = \frac{2.5\lambda}{22} \tag{13}$$

5. FC energy function

The fuel cell stack state space model is now given using the electrical equivalence and analogy between fluidic-electric and pneumatic-electric. A major advantage of having an electrical equivalent circuit of FC stack is the possibility to define the natural energy function of the stack.

The energy function is always defined according to the energy storage devices. Hence, the natural energy is:

$$H(P) = \frac{1}{2}C_f^a(P_{C_f}^a)^2 + \frac{1}{2}C_f^m(P_{C_f}^m)^2 + \frac{1}{2}C_f^c(P_{C_f}^c)^2 \tag{14}$$

In Eq. (14), three energies are defined according to the stored energy in anode, membrane and cathode. The proposed energy function can be evaluated since the information about the three different pressures (anode, membrane and cathode) can be easily measured. This energy function plays a central role in the FC energy management by defining a desired energy function according to the desired anode, membrane and cathode pressures. It also helps in control based applications as shaping the natural FC stack energy towards the desired one. Automatic control techniques like passivity-based control, energy balance or energy shaping techniques are some promising new techniques which offer a possible solution to this problem [12,13]. The definition of the desired anode, membrane and cathode pressures (and consequently the desired energy function) can be done with respect of different criteria as:

Table 1
Operating conditions for the FC stack.

Number of single cells	20
Operating temperature (°C)	20–65
Nominal temperature (°C)	55
Cell area (cm^2)	100
Operating pressure	From 1 to 1.5 bar abs
Differential pressure between anode and cathode compartments	Up to 0.6 bar
Inlet media	Humidified air Pure hydrogen De-ionized cooling water

- Membrane humidification;
- Reducing intra-FC stack energy losses;
- Producing enough output energy to match the load demands;
- Maximization of the FC stack lifespan;
- . . .

This list of criteria is not exhaustive and can primarily be defined with the help of the practitioners. Multi-criteria optimization technique could also help to define the desired (optimal) energy function.

Another interesting representation is the port controlled Hamiltonian representation (PCH) which exhibits important physical properties like variables interconnection and the system damping. Authors are working to find out this PCH form and results should be presented in a future paper.

6. FC test benchmark description

The manufactured PEMFC used in this study has been assembled with commercially available membranes (Gore MESGA Primea Series 5510), diffusion layers and machined graphite flow distribution plates. Data on the FC as well as nominal operating conditions are given in Table 1. A detailed description of the 1 kW test facility can be found in [7,8]. The test bench has been designed to test up-to 1 kW fuel cells in different operating modes (flow-through, dead-end mode) and for different operating parameters (back pressure, gas temperature, operating temperature, gas utilization ratio, humidification rates for incoming air and hydrogen, electric load). The facility gathered although all the safety equipment required by the use of hydrogen.

Many physical parameters involved in the stack can be controlled and measured in order to control the FC operating conditions as accurately as possible. Moreover, inlet and outlet flows, pressures, temperatures, single cell voltages, and current can be monitored. A picture representing the considered fuel cell stack on the test bench is given in Fig. 6, this test bench is available in the FCLab (Fuel Cell Lab.), Belfort, France.

7. Simulation and experimental validation

The model we have developed in the earlier section needs to be verified with the experimental data. We chose different sets of experimental data to check the robustness of this model. The simulations have been carried out in Matlab-Simulink. Figs. 7–9 respectively show the simulations for a 400 W, 500 W and a 700 W PEM fuel cell stack having 20 cells. The simulated voltage is con-

Fig. 6. Fuel cell benchmark setup.

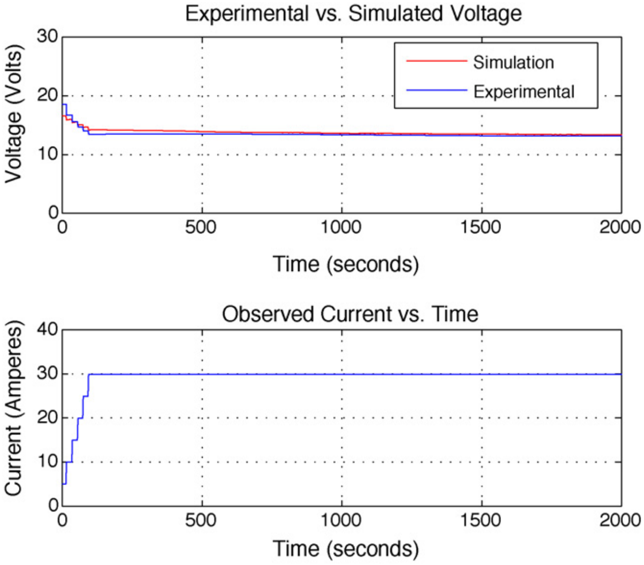

Fig. 7. Simulated and experimental voltages obtained for a 400 W PEMFC stack.

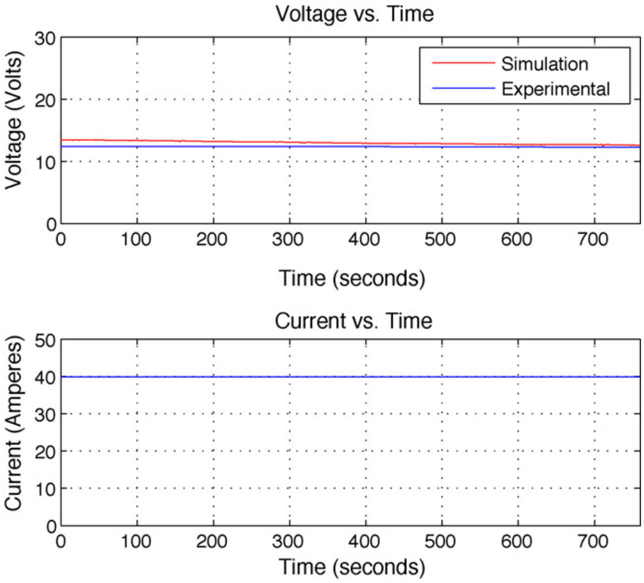

Fig. 8. Simulated and experimental voltages obtained for a 500 W PEMFC stack.

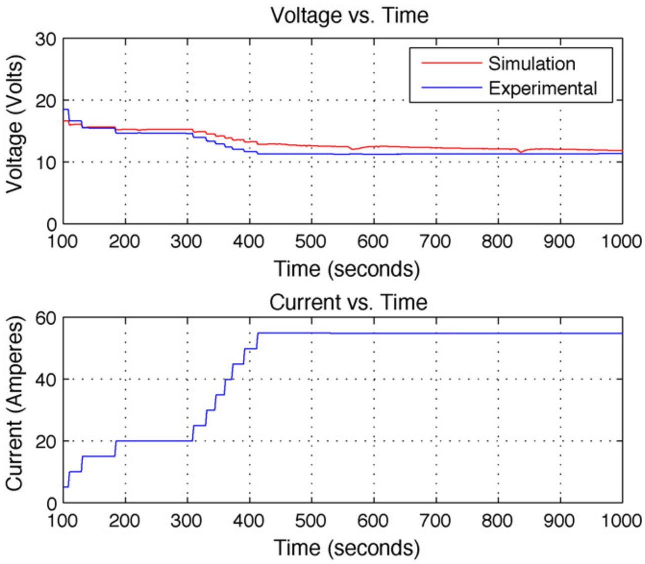

Fig. 9. Simulated and experimental voltages obtained for a 700 W PEMFC stack.

siderably close to the voltage obtained experimentally in both the cases. The subplots in each figures represent the FC current imposed by the load and playing the role of a perturbation for FC.

The experimental test done for PEMFC of 500 W is a static test, while the ones done for the PEMFC of 400 W and 700 W have step changes in the load current.

The simulated voltage is considerably close to the voltage obtained experimentally in both the cases. The error for the three tests is less than 10%. Note that the maximum error is generally observed for a very small time particularly at the time of initiation up to 100 s of operation. The error in remaining part is observed to be less than 3% which also tends to reduce further with time. However, we see a slight offset in the values, which is due to the non-steady pneumatic resistances. The reader may be inclined to think if there could be a correction factor which can be introduced to reduce the offset. Such an approach has been considered by the authors. The use of a correction factor although, would bring the offset values (within 3% error limits) even more close to desirable values with lesser error, but would result in higher error value during the initial phase of operation. The parameters calculated are under the specific zones of operation and the deviation can be accounted for by the migration of the operating zones away from the zones that are used for parameter calculations. Such a migration would render these parameters unfit and call for their recalculation and hence require the calculation of these parameters again. Obtaining the exact values/relations for the parameters requires rigorous computations of a wide range of experimental data available and discussion of which is beyond the scope of this text.

8. Conclusion

A simple and reliable model for a FC stack suitable for application in electrical engineering and automatic control was developed. This model was obtained using a careful analogy between the pneumatics/fluidics in the FC and their appropriate electrically equivalent components. The effects of temperature and humidity were taken into consideration, variation on the volume of water condensed as well as the storage capacity of channels which are modeled by capacitors, whose values have also been carefully determined. The simulated model was validated using experimental data by conducting the experiments in the fuel cell laboratory, FCLab, France.

The proposed PEMFC stack model consists of only three order state space equations. The rest of the equations are simple static ones, but dependant on the temperature and humidity. This original model can be of a great use in automatic control and energy management scopes. Another key point is the definition of the natural PEMFC stack energy function which can be of great interest in the control and energy management tasks. The simulation results are very close to the experimental ones; for the three tests (400 W, 500 W and 700 W FC) a maximum of 10% error is observed in the initial phase of operation particularly up to 100 s. The maximum error in the remaining part is observed to be less than 3% and constantly reducing as time increases. Nevertheless, this light deviation can also be directly linked to the difficulty of obtaining enough accurate estimation of the parameters values, especially those linked to the internal behaviour of the FC stack. If the user of the model is able to obtain more accurate values, the resulting error could be strongly reduced.

The figures presented in the previous section give us a clear idea of the usability of this model. The model achieves a good stack voltage prediction. The possible causes for deviation in the simulated and experimental values have also been presented. Besides formulating a model, this paper also opens up the possibility of making this model more inclusive especially of the processes which have been treated to be the part of the auxiliary systems for the sake of keeping this model simple. There is a lot of scope for building up on this model further, which would be taken as a subject of subsequent research.

Appendix A.

Note that R is an equivalent resistance and is not a real electrical resistance.

All the parameters used in the equations obtained from the circuit can be tabulated as follows:

Variable	Description	Units
Q^c_{in}	Inlet flow rate at cathode	$\mathrm{l\,min^{-1}}$
$Q^I_{w,r}$	Rate of formation of water in reaction, function of current	$\mathrm{mol\,min^{-1}}$
$Q^I_{O,r}$	Rate of consumption of oxygen in reaction, function of current	$\mathrm{mol\,min^{-1}}$
$Q^c_{P_{sat}}$	Flow rate of condensed water on cathode side	$\mathrm{l\,min^{-1}}$
$Q_{N,m}$	Flow rate of nitrogen via membrane	$\mathrm{l\,min^{-1}}$
$Q_{w,m}$	Flow rate of water through membrane	$\mathrm{l\,min^{-1}}$
Q^c_{out}	Outlet flow rate at cathode	$\mathrm{l\,min^{-1}}$
Q^a_{in}	Inlet flow rate at anode	$\mathrm{l\,min^{-1}}$
$Q^m_{C_f}$	Flow due through capacitance in membrane	$\mathrm{l\,min^{-1}}$
$Q^c_{C_f}$	Flow through capacitance on cathode side	$\mathrm{l\,min^{-1}}$
$Q^I_{H,r}$	Rate of consumption of hydrogen in reaction	$\mathrm{mol\,min^{-1}}$
$Q^a_{P_{sat}}$	Flow rate of condensed water on anode side	$\mathrm{l\,min^{-1}}$
Q^a_{out}	Outlet flow rate at anode	$\mathrm{l\,min^{-1}}$
$P^c_{C_f}$	Pressure across cathode capacitance	mbar
P^c	Pressure at cathode outlet	mbar
$P^a_{C_f}$	Pressure across cathode capacitance	mbar
P^a	Pressure at anode outlet	mbar
P^c_{in}	Pressure at cathode inlet	mbar
$P^m_{C_f}$	Pressure across membrane capacitance	mbar
R^a_{out}	Resistance at anode outlet	$\mathrm{mbar\,min\,l^{-1}}$
R^a_{ch}	Resistance of anode channel	$\mathrm{mbar\,min\,l^{-1}}$
R^c_{out}	Resistance at cathode outlet	$\mathrm{mbar\,min\,l^{-1}}$
R^c_{ch}	Resistance of cathode channel	$\mathrm{mbar\,min\,l^{-1}}$
R^c_{in}	Resistance at cathode inlet	$\mathrm{mbar\,min\,l^{-1}}$
$R_{w,m}$	Membrane resistance to water	$\mathrm{mbar\,min\,l^{-1}}$
$R_{N,m}$	Membrane resistance to nitrogen	$\mathrm{mbar\,min\,l^{-1}}$
C^c_f	Cathode capacitance	$\mathrm{l\,mbar^{-1}}$
C^a_f	Anode capacitance	$\mathrm{l\,mbar^{-1}}$
C^m_f	Membrane capacitance	$\mathrm{l\,mbar^{-1}}$
$Q^I_{w,r-e}$	Flow rate of water due to reverse osmosis, function of current	$\mathrm{mol\,min^{-1}}$
ν_w	Molar volume of water	$\mathrm{l\,mol^{-1}}$
ν_o	Molar volume of oxygen	$\mathrm{l\,mol^{-1}}$
ν_h	Molar volume of hydrogen	$\mathrm{l\,mol^{-1}}$

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